Integrated System Identification and State Estimation for Control of Flexible Space Structures

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A novel approach is developed for identification of a state-space model and a Kalman filter gain from input and output data. There are four steps involved in this approach. First, the relation between a stochastic state-space model of a dynamical system and the coefficients of its autoregressive model with exogeneous input is derived. Second, an adaptive least-squares transversal predictor is used to estimate the coefficients of the model. Third, a state-space model and a steady-state Kalman filter gain of the dynamical system are then identified from the coefficients of the model by using the eigensystem realization algorithm. Fourth, a modal state estimator is constructed using the modal parameters of the identified model. On-line implementation of this algorithm can continually improve the modal parameters and the filter gain. It can also gradually update the system model when the system characteristics are slowly changing. Numerical examples are used to illustrate the feasibility of the new approach.

Introduction

FOR active control of large flexible space structures, accurate models and accurate state information of the systems are very important. A control design generally demands an accurate model that adequately describes the dynamic behavior of the system, and if state feedback is used, accurate state information is required to determine the control force. The state estimator or filter for a stochastic system is needed for two reasons. First, the number of sensors used to measure the vibrational motion of a structure is usually less than the number of the states of interest. Second, the system is generally disturbed by unknown process noises and the measurement data are always corrupted by measurement noises. The process noises arise from such factors as unknown excitations and unmodeled nonlinearities. The measurement noises are usually additive due to imperfect instruments. Good performance of a state estimator relies both on an accurate system model and an accurate estimate of the noise statistics (or of the optimal filter gain).

For a large flexible space structure, because of its great flexibility and gravitational load, an accurate system model cannot be obtained from ground testing. In addition, the system characteristics of a large flexible structure in space can vary due to such factors as temperature gradient induced by shadowing, reorientation of a large antenna, or deterioration of material. The system model needs to be updated frequently. Hence, for better control performance, on-line adaptive system identification is required. In Ref. 1, lattice filter theory was used to recursively identify the coefficients of an autoregressive moving-average (ARMA) model; however, the method of computing the state-space model parameters from the ARMA model is not mentioned. The eigensystem realization algorithm (ERA) has been proved effective in modal parameter identification and model reduction of dynamic systems from test data.[2-7] However, ERA uses free decay or impulse response data, which may not be suitable for on-orbit implementation.

Kalman filter is a widely used technique in state estimation.[8] However, a well-known limitation in applying Kalman filter is the requirement of a priori knowledge about the system state-space model and the statistics of the disturbing noises. This information, in practice, is either partially known or totally unknown. Adaptive filtering, aimed at removing this limitation, can adaptively identify the noise statistics during the filtering process.[9-11] However, most of the adaptive filtering methods are derived under the assumption that the system state model is known a priori. Adaptive filtering for uncertain systems is seldom addressed.

In this paper, a novel approach that integrates system identification and state estimation is developed for identification of a state-space model and a Kalman filter gain from input and output data. The approach includes four steps. First, a state-space model is used to explicitly relate the Kalman filter formulation to an ARX model, a special form of ARMAX (auto regressive moving average model with exogeneous input). Second, an adaptive transversal predictor is used to identify the coefficients of the ARX model from input/output data. The advantage of using the ARX model is that its coefficients can be obtained adaptively starting from zero initial values by a standard recursive least-squares (RLS) method, which is commonly used in the field of signal processing.[12] Furthermore, if the fading memory technique is used, the weighted recursive least-squares method can estimate the

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coefficients of a slowly time-varying system.[12] The estimation of the coefficients is unbiased and p consistent in the sense that the estimate converges almost surely as the number of data tends to infinity, which in turn converges to the true coefficients as the degree p approaches infinity. Third, from the estimated coefficients of the ARX model, ERA is used to realize a state-space model and a steady-state Kalman filter gain. Fourth, the realized matrices are transformed to the modal space, and a modal state estimator is derived. From the modal space, the modal frequencies, modal damping, and mode shapes of the system can be identified.[3] Numerical examples are given to illustrate and validate the method developed in this paper.

Relation Between State-Space and ARX Models

A finite-dimensional, linear, discrete, time-invariant stochastic dynamic system can be represented by a state-space model as

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where x is an $n \times 1$ state vector, u an $m \times 1$ input vector, and y a $p \times 1$ measurement or output vector. Matrices A, B, and C are the state matrix, input matrix, and output matrix, respectively. The sequence $\{w_k\}$ is the process (input) noise, assumed to be Gaussian, zero meaned, and white with covariance matrix Q. The sequence $\{v_k\}$ is the measurement (output) noise, also assumed to be Gaussian, zero meaned, and white with covariance matrix R. The integer k is the sample indicator. The sequences $\{w_k\}$ and $\{v_k\}$ are assumed statistically independent of each other.

The same system can also be represented by a stochastic ARMAX model,

$$A(q^{-1})y_k = B(q^{-1})u_k + C(q^{-1})e_k \tag{3}$$

where

$$A(q^{-1}) = I_p + A_1q^{-1} + \cdots + A_{na}q^{-na} \tag{4}$$

$$B(q^{-1}) = B_1q^{-1} + \cdots + B_{nb}q^{-nb} \tag{5}$$

$$C(q^{-1}) = I_p + C_1q^{-1} + \cdots + C_{nc}q^{-nc} \tag{6}$$

q^{-1} is a backward shift operator (i.e., $q^{-1}y_k = y_{k-1}$), I_p a $p \times p$ identity matrix, and na, nb, nc are the orders of the polynomials $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$, respectively. The sequence $\{e_k\}$ is a white noise process with zero mean. The term $A(q^{-1})y_k$ is the autoregressive (AR) part, $C(q^{-1})e_k$ the moving average (MA), and $B(q^{-1})u_k$ the exogeneous variable (X) part.

The state-space model provides inner messages about the states of the system in addition to the input-output information, whereas the ARMAX model gives the relation between the input and the output only. These two different models describe the same system; hence, they must be related. Indeed, the relation can be made through the Kalman filter. One can write the following filter innovation model, which describes the filter system as driven by the innovation sequence, and the output are the measurement data

$$\hat{x}_{k+1}^- = A\hat{x}_k^- + Bu_k + AK_k\epsilon_k \tag{7}$$

$$y_k = C\hat{x}_k^- + \epsilon_k \tag{8}$$

where $\hat{x}_k^-$ is the best estimate of the state x_k in the sense of least mean square error, based on the data $\{y_{k-1}, \ldots, y_0, u_{k-1}, \ldots, u_0\}$. The term ϵ_k is called residual and is defined as the difference between the real measurement y_k and the predicted measurement, $C\hat{x}_k^-$. The quantity ϵ_k contains the new information in the sense that it cannot be obtained from the previous data. Therefore, it is also called innovation.[13] The $n \times m$ matrix K_k is the Kalman filter gain. Introducing Eq. (8) into Eq. (7) yields

$$\hat{x}_{k+1}^- = A(I_n - K_kC)\hat{x}_k^- + Bu_k + AK_ky_k \tag{9}$$

where I_n is the $n \times n$ identity matrix.

The existence of the steady-state Kalman filter gain K is guaranteed if the system is detectable and $(A, Q^{1/2})$ is stabilizable.[11] In the implementation of the Kalman filter, one can start from an arbitrary guess of the initial state value and its corresponding error covariance. For a stable filter, the Kalman filter gain will converge exponentially to its steady-state value independently of the initial condition.

From Eqs. (8) and (9) with the steady-state Kalman filter gain K, one can obtain the following input/output description:

$$\begin{aligned} y_k &= CAKy_{k-1} + C\bar{A}AKy_{k-2} + \cdots + C\bar{A}^{M-1}AKy_{k-M} \\ &\quad + CBu_{k-1} + C\bar{A}Bu_{k-2} + \cdots + C\bar{A}^{M-1}Bu_{k-M} \\ &\quad + \epsilon_k + C\bar{A}^M\hat{x}_{k-M}^- \\ &= \sum_{i=1}^{M} C\bar{A}^{i-1}AKy_{k-i} + \sum_{i=1}^{M} C\bar{A}^{i-1}Bu_{k-i} + \epsilon_k \\ &\quad + C\bar{A}^M\hat{x}_{k-M}^- \end{aligned} \tag{10}$$

for some integer M and

$$\bar{A} = A(I_n - KC) \tag{11}$$

Note that, although the steady-state Kalman filter gain might not be known at the very beginning, it has already existed. This implies that Eqs. (10) are a valid relation even for the very beginning data. In other words, once the value of every input/output term in Eqs. (10) is known, these equations hold. Note that $\bar{A}$ in Eq. (11) is the system matrix of the filter dynamical equation, Eq. (9), and also is the system matrix of the filter error dynamical system.[14] For a stable filter, the matrix $\bar{A}$ is asymptotically stable. Therefore, the last term of Eqs. (10) is negligibly small and can be neglected for a sufficiently large number M. Moving all of the terms containing output y to the left side, Eqs. (10) become

$$y_k - \sum_{i=1}^{M} C\bar{A}^{i-1}AKy_{k-i} = \sum_{i=1}^{M} C\bar{A}^{i-1}Bu_{k-i} + \epsilon_k \tag{12}$$

which is a special form of an ARMAX model with $C(q^{-1}) = I_p$, hence called ARX because it has no moving average part. All of the coefficients of this model are expressed in terms of the system parameters A, B, C and the Kalman gain K. Note that the noise term here is the residual of the optimal Kalman filter, which is zero meaned, white, and not correlated with previous output data according to the orthogonality principal in estimation.

Estimation of the Coefficients of the ARX Model

Estimation of the coefficient matrices of the ARX model given in Eq. (12) can be accomplished by using an adaptive transversal predictor (filter).[12,15] This recursive least-squares filter, as shown in Fig. 1, sequentially feeds the input/output measurements and their delay versions to its M tap inputs for a filter of order M. Each tap input signal is multiplied by the tap coefficient matrix and the results are summed to yield the filter output, which is the one-step-ahead output prediction. The prediction error is then fed back to modify the tap coefficient matrices in the next recursion.

Equation (12) can be written as

$$\begin{aligned} y_k &= \sum_{i=1}^{M} C\bar{A}^{i-1}AKy_{k-i} + \sum_{i=1}^{M} C\bar{A}^{i-1}Bu_{k-i} + \epsilon_k \\ &= \hat{y}_k + \epsilon_k \end{aligned} \tag{13}$$

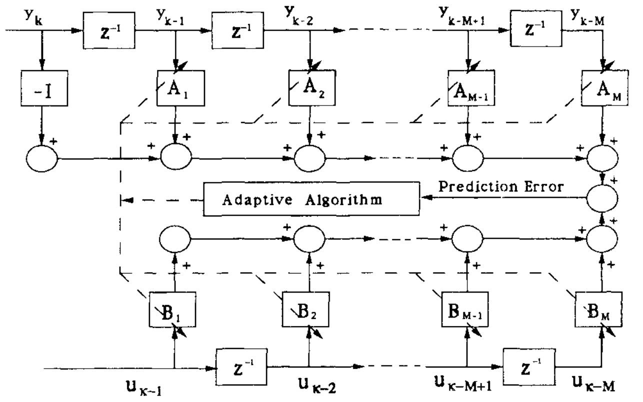


Fig. 1 Adaptive transversal predictor.

which can be interpreted in two different ways. On one hand, it can be regarded as a signal generator, where y_k is synthesized by using finite previous input/output data $\{u_{k-1}, \ldots, u_{k-M}, \ldots, y_{k-1}, \ldots, y_{k-M}\}$ and white noise ϵ_k. On the other hand, it can also be viewed as a linear predictor, where $\hat{y}_k$ is a prediction of y_k and ϵ_k is the prediction error. The output y_k can be thought of as truly coming from a linear transversal process generator driven by the known deterministic input and unknown white noise. From adaptive filter theory, the reverse process of feeding the output and input sequences $\{y_k\}$ and $\{u_k\}$ to a transversal process analyzer (transversal filter) can asymptotically retrieve the white noise process.[12] If the statistics of input/output data are not known, an adaptive transversal filter can be used, and the tap coefficient matrices will converge. If the noise is zero meaned and not correlated with the input/output data, which is the case for Eqs. (13), the estimation of the coefficient matrices is unbiased and p consistent,[16] i.e., as the number of data tends to infinity, the estimate converges almost surely to a matrix, which, in turn, converges to the true parameter matrix as the degree of the ARX model tends to infinity. The degree of the ARX model in the original paper[16] is denoted by p and, hence, the name.

In implementing the transversal predictor, its order should be determined in advance. It should be sufficiently large in order to yield a satisfactory result. A method based on the Akaike information-theoretic criterion (AIC) is commonly used.[17,18] The computational load increases rapidly as the filter order increases for the conventional least-squares filter. However, by using the fast transversal filters (FTF) or the recursive least-squares lattice filters (LSL), the computational cost can be greatly reduced to be only linearly proportional to the increase of the filter order.[12]

From the adaptive transversal predictor, two sets of coefficient matrices are obtained:

$$S_1 = \{CAK, C\bar{A}AK, \ldots, C\bar{A}^{M-1}AK\} \tag{14}$$

$$S_2 = \{CB, C\bar{A}B, \ldots, C\bar{A}^{M-1}B\} \tag{15}$$

Sets S_1 and S_2 in Eqs. (14) and (15) contain members that have the same form as the Markov parameters. In fact, S_1 is the Markov parameters of the filter system described by Eq. (9) when driven by y_k, while S_2 is the Markov parameters when driven by u_k. The method of realizing the model matrices A, B, C, and the steady-state Kalman filter gain K from these two sets is described in the following section.

Realization of a State-Space Model and Steady-State Filter Gain via Eigensystem Realization Algorithm

The ERA is a simple and accurate algorithm for identification of linear systems from impulse responses. It has been proved valuable for modal parameter identification from test data.[7] The algorithm uses the impulse responses (i.e., Markov parameters) to form a large block data matrix, which is referred to as the general Hankel matrix. Then, the technique of singular value decomposition is used to decompose the Hankel matrix. The system order is determined by counting the number of singular values retained. The small singular values are attributed to noises and are truncated. The state-space model can be computed from the decomposed matrices. The realized model is not unique, but the Markov parameters are unique. For further details, the readers are referred to Ref. 3.

The following procedure provides a realization of the state-space model A, B, C, and the steady-state Kalman filter gain K. First, the ERA is used to decompose the set S_1 to obtain a realization $[\bar{A}', (AK)', C']$, which is related to $[\bar{A}, AK, C]$ via some equivalent transformation P. Specifically,

$$\bar{A}' = P^{-1}\bar{A}P \tag{16}$$

$$(AK)' = P^{-1}(AK) \tag{17}$$

$$C' = CP \tag{18}$$

Note that, with respect to the equivalent transformation P, a realized output matrix C' is readily available as given in Eq. (18). The corresponding realized system matrix $A' = P^{-1}AP$ can be obtained from Eqs. (11) and (16–18) as

$$A' = \bar{A}' + (AK)'C' \tag{19}$$

To obtain the realized input matrix under the same transformation, namely, $B' = P^{-1}B$, first note that

$$CB = CPP^{-1}B = C'B'$$

$$C\bar{A}^iB = CPP^{-1}\bar{A}^iPP^{-1}B = C'(\bar{A}^i)'B'$$

$$= C'(\bar{A}')^iB'$$

Hence, from the set S_2 one can write

$$\begin{bmatrix} CB \\ C\bar{A}B \\ \vdots \\ C\bar{A}^{M-1}B \end{bmatrix} = \begin{bmatrix} C'B' \\ C'\bar{A}'B' \\ \vdots \\ C'(\bar{A}')^{M-1}B' \end{bmatrix} = \begin{bmatrix} C' \\ C'\bar{A}' \\ \vdots \\ C'(\bar{A}')^{M-1} \end{bmatrix} B'$$

$$= VB' \tag{20}$$

where V denotes the observability-like matrix. Thus, B' can then be calculated as

$$B' = V^{\dagger} \begin{bmatrix} CB \\ C\bar{A}B \\ \vdots \\ C\bar{A}^{M-1}B \end{bmatrix} \tag{21}$$

where $V^{\dagger}$ denotes the pseudoinverse of V.

An alternative way to obtain matrix B, which is used in the numerical example, is to apply ERA to the set S_2. However, the realized matrix B is not necessary in the same coordinates as the realized system matrix A' and output matrix C' obtained using the set S_1. A coordinate transformation is needed to bring the two sets of identified matrices to the same coordinates, which will be discussed in the next section.

Under different coordinates for the state variable, the steady-state Kalman gain should be transformed accordingly. If the state is transformed by a nonsingular matrix P, i.e., $\hat{x}_k^- = P(\hat{x}_k^-)'$, then Eq. (7) becomes

$$(\hat{x}_{k+1}^-)' = P^{-1}AP(\hat{x}_k^-)' + P^{-1}Bu_k + P^{-1}APP^{-1}K\epsilon_k$$

$$= A'(\hat{x}_k^-)' + B'u_k + A'K'\epsilon_k$$

where $K' = P^{-1}K$. Note that K is transformed exactly the same way as B. Since $(AK)' = A'K'$, K' can be calculated as

$$K' = (A')^{-1}(AK)' \tag{22}$$

Modal State Estimation

For a given realized quadruplet $[A', B', C', K']$, a state estimator for the original system can be constructed using this quadruplet. Because the sets S_1 and S_2 can be adaptively improved when more data are processed, ERA must be used from time to time to update the quadruplet. However, each time the quadruplet may not necessarily belong to the same coordinates. Therefore, in order to compare successively the realized quadruplet and estimated states, they should be transformed to the same coordinates. The modal coordinate, where the system matrix A is block diagonal and the matrices C and B are normalized in some sense, is an appropriate choice.

Consider the case when all eigenvalues of the system matrix A are distinct. Then, the normalized eigenvector matrix $V = [v_1, v_2, \ldots, v_n]$ can be used to diagonalize A, i.e., $V^{-1}AV = \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, where λ_i $(i = 1, 2, \ldots, n)$ denotes the ith eigenvalues of A. Since a scalar multiple of an eigenvector is still an eigenvector, any $T = VK_c$ can also diagonalize A, where K_c is any (nonsingular) diagonal matrix. Furthermore, any matrix T that diagonalizes A can be written as $T = VK_c$ for some K_c. A three-step procedure is presented in the following to transform a realized triplet $[A, B, C]$ to its modal coordinates.

Step 1: Diagonalization

Let the system matrix A be diagonalized by a matrix T such that

$$A^* = T^{-1}AT = \Lambda \tag{23}$$

Correspondingly, B and C are transformed according to

$$B^* = T^{-1}B, \qquad C^* = CT \tag{24}$$

Given two sets of realized $[A, B, C]$ that are equivalent, i.e., related by some equivalent transformation, the above transformation, Eqs. (23) and (24), will uniquely recover $A^* = \Lambda$ but not necessarily B^* and C^* because of the freedom in T. In order to uniquely recover B^* and C^*, they must be normalized in a certain way. The following describes such a normalization procedure.

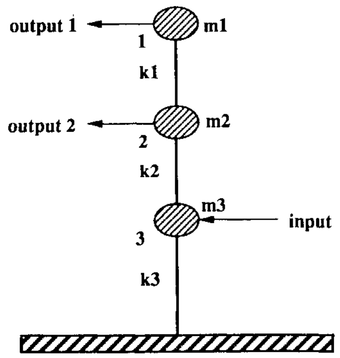
Step 2: Normalization

The normalization is defined such that each column of the normalized matrix has unit length and the first element of the column is a positive real number. Noting that the elements could be complex numbers, this procedure can be accomplished by the following steps. First, find a constant diagonal matrix M, $M = \text{diag}[m_1, m_2, \ldots, m_m]$, such that $C^*M = [c_1m_1, \ldots, c_nm_n]$ and $m_i^2 c_i^T c_i = 1$, where c_i denotes the ith column of C^*. Next, find a constant diagonal matrix R, $R = \text{diag}[r_1, \ldots, r_n]$, where r_i is a pure complex number or $+1(-1)$ which rotates a complex number without changing its length, such that $C_n = C^*MR = \text{diag}[c_1m_1r_1, \ldots, c_nm_nr_n]$ and the first element of vector $c_im_ir_i$ is a positive real number. Then C^* is normalized such that

$$C_n = C^*MR \tag{25}$$

Accordingly, B^* is transformed to B_n such that

$$B_n = R^{-1}M^{-1}B^* \tag{26}$$



m1	0.5
m2	1.0
m3	1.0
k1	10.0
k2	10.0
k3	10.0

Fig. 2 Simulated lumped-mass beam.

Step 3: Modal Transformation

For vibratory systems such as flexible space structures, the eigenvalues often appear as complex conjugate pairs. After steps 1 and 2 the states are complex numbers, and so are the elements of B_n and C_n. Another transformation can be used to further transform $[\Lambda, B_n, C_n]$ to their modal forms, $[A_m, B_m, C_m]$, where A_m is block diagonal and all the matrices are real. The realized steady-state Kalman filter gain K is transformed in the same way as B.

It can be shown that the above transformation procedure described in steps 1–3 will recover a unique set of $[A_m, B_m, C_m, K_m]$ from any equivalent sets of $[A, B, C, K]$ (see Appendix A for proof). With the quadruplet in modal form, the modal space state estimation can be carried out by using a constant gain Kalman filter. The modal state estimator is described by the following two equations:

$$\hat{x}_k^- = A_m\hat{x}_{k-1}^- + B_m u_{k-1} \tag{27}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_m(y_k - C_m\hat{x}_k^-) \tag{28}$$

where $\hat{x}_k^+$ is the estimated state. Note that A_m is a block diagonal matrix that makes this state estimator easier for implementation. To this end, the integrated system identification and state estimation scheme has been accomplished.

Numerical Examples

To study the numerical properties of the algorithm, two sample problems are presented. In the first example, the system is a beamlike structure characterized by nonrepeated low frequencies and low dampings. In the second example the system is a simulated Mini-Mast structure characterized by repeated low frequencies and low dampings.

Table 1 Comparison of the identified modal parameters of example 1

Number of data processed	Mode 1 Frequency, rad/s	Mode 1 Damping, %	Mode 2 Frequency, rad/s	Mode 2 Damping, %	Mode 3 Frequency, rad/s	Mode 3 Damping, %
0[a]	1.6369	0.63	4.4719	1.01	6.1085	1.30
2000	1.6711	16.06	4.5988	5.14	10.6371	4.15
3000	1.6920	6.59	4.5118	3.79	7.0251	11.29
4000	1.6405	5.22	4.4252	1.69	6.5016	5.72
5000	1.6245	4.73	4.4139	0.89	6.3176	4.75

[a]True values.

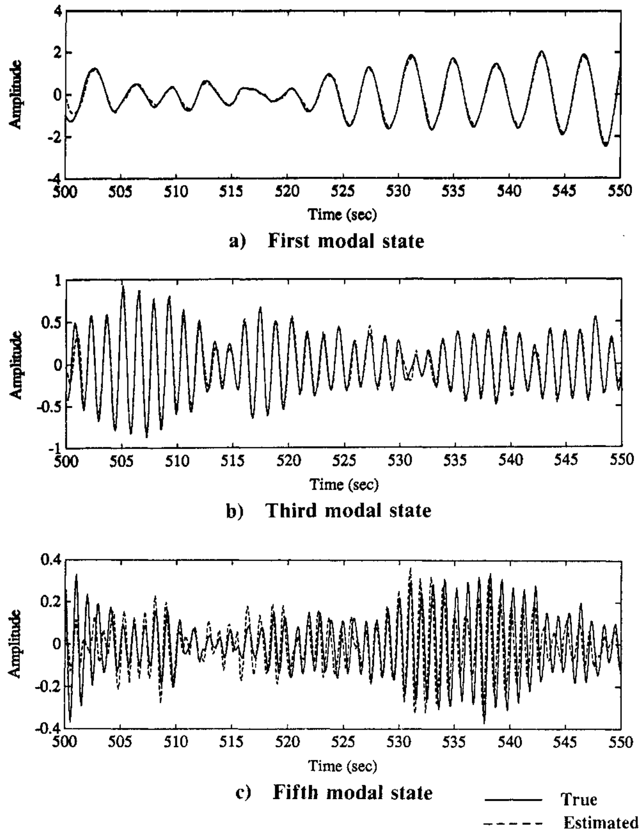


Fig. 3 State estimation after 5000 data processed.

Example 1: Beamlike Structure

In the first example, the lumped-mass beamlike simulated system with three masses (see Fig. 2) has three modes (six states). The system is excited by random force u at node 3, and the responses are measured at nodes 1 and 2. The modal frequencies and damping factors are shown in Table 1. The variance of the random excitation force σ_u^2 is set to 40. The standard deviation of the process noise w_k is about 23% of that of the input influence Bu_k. The standard deviation of the measurement noise v_k is about 10% of that of the output measurement y_k. The sampling frequency is 10 Hz, which is sufficient for identification of the highest frequency (0.97 Hz) of the system. The filter order of the adaptive transversal filter is set at 100. The identified modal frequencies and damping factors after processing 2000, 3000, 4000, and 5000 data are listed in Table 1 and compared with the true values. The true quadruplet $[A_m, B_m, C_m, K_m]$ is shown in Appendix B along with its identified version $[\hat{A}_m, \hat{B}_m, \hat{C}_m, \hat{K}_m]$ obtained after processing 5000 data. Note the true and identified quadruplets are in fairly good agreement. Theoretically, as more data are processed, these parameters converge to their true values if the filter order is sufficiently large.

Using the identified quadruplet $[\hat{A}_m, \hat{B}_m, \hat{C}_m, \hat{K}_m]$, a modal state estimator is constructed as shown in Eqs. (27) and (28). The estimated modal states are then compared with their true values. Figures 3 show the true (solid line) and estimated (dashed line) modal state histories using the identified quadruplet with 5000 data processed. Three modal state histories are shown in Figs. 3. Note that although the modal parameters have not quite converged yet, the state estimations are fairly accurate. Figure 4 shows the plot of true and estimated values of the (1,1) elements of the matrices $C\bar{A}^{i-1}AK(i = 1, \ldots, 100)$ vs power index number i after processing 5000 data. The circle marks represent the true values and the star marks the identified ones. Note that the sequence in Fig. 4 corresponds to a Markov parameter element [see Eq. (14)] of the filter system dynamics. Convergence of the identified filter Markov parameters is important in identifying the filter dynamics and

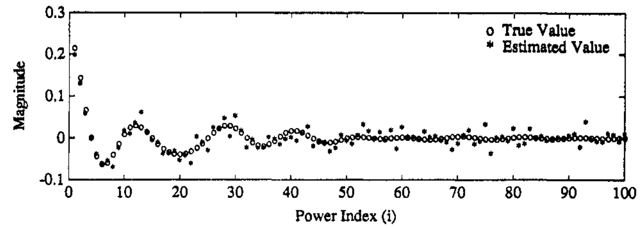


Fig. 4 Comparison of an estimated filter Markov parameter sequence [the (1,1) element of matrices $C\bar{A}^{i-1}AK$] and its true value after 5000 data processed.

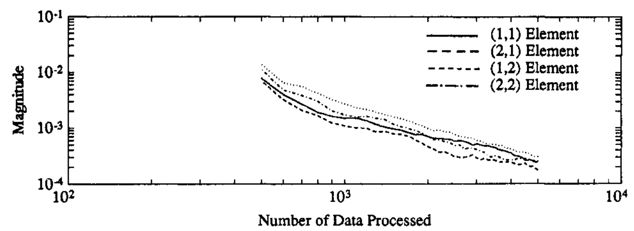


Fig. 5 Error variances of the estimated filter Markov parameter sequence (element of matrices $C\bar{A}^{i-1}AK$) vs number of data processed.

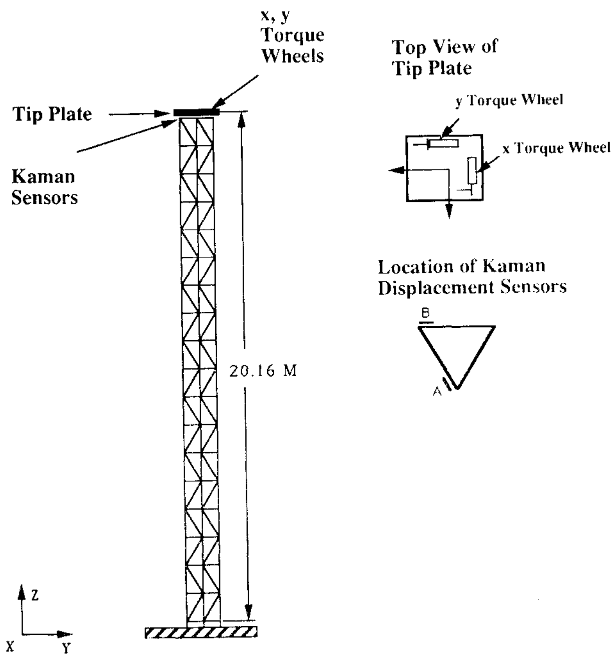


Fig. 6 Mini-Mast structure.

the system itself. Figure 5 shows convergence of the error variances of the identified parameter sequences (elements of matrices $C\bar{A}^{i-1}AK$) vs the number of data processed. Note that the algorithm does not make parameter estimation until it receives a sufficient amount of data. The rate of convergence is reasonably good.

Example 2: Mini-Mast

In the second example, a simulated Mini-Mast model is considered. Mini-Mast is a 20-m-long generic space truss built in NASA Langley Research Center for control experiments of flexible structures[19] (see Fig. 6). It is deployed vertically and cantilevered from its base on a rigid foundation. A five-mode (10 states) model of Mini-Mast, which includes two repeated frequencies, is used to generate simulated data. The modal frequencies and damping factors of the model are listed in Table 2. The first two modes are closely spaced, representing the first bending mode in the x and y axes (see Fig. 6) with the same mode shapes in different phases, and similar are the last two closely spaced modes, which are the second bending

Table 2 Comparison of the identified modal parameters of example 2

Number of data processed	Mode 1 Frequency, rad/s	Mode 1 Damping, %	Mode 2 Frequency, rad/s	Mode 2 Damping, %	Mode 3 Frequency, rad/s	Mode 3 Damping, %	Mode 4 Frequency, rad/s	Mode 4 Damping, %	Mode 5 Frequncy, rad/s	Mode 5 Damping, %
0[a]	5.0318	1.80	5.0356	1.80	27.4201	1.20	38.3511	1.00	38.6823	1.00
10,000	5.0358	1.95	5.1846	5.86	27.5108	2.35	38.3804	0.79	96.7756	2.39
13,000	5.2096	2.19	5.1484	3.78	27.4360	1.33	38.2289	1.06	38.3230	2.43
15,000	5.0463	1.87	5.1310	3.36	27.3502	0.53	38.0128	1.51	38.4156	1.15

[a]True values.

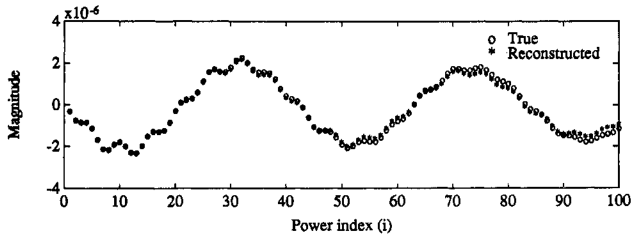
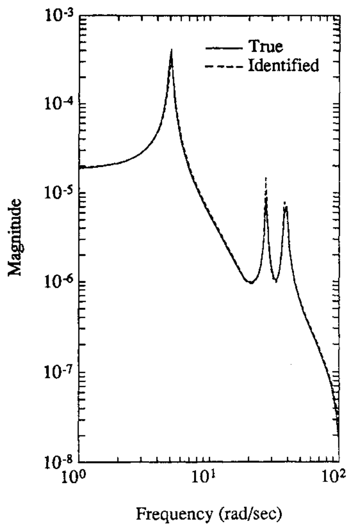
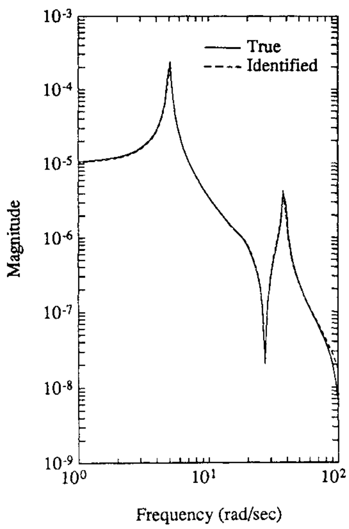


Fig. 7 Comparison of a reconstructed system Markov parameter sequence [the (1,1) elements of $\hat{C}\hat{A}^{i-1}\hat{B}$] and its true value after 15,000 data processed.



a) First singular value



b) Second singular value

Fig. 8 Comparison of the singular value frequency response.

modes. The third mode represents the first Mini-Mast torsion mode. The simulated system has two inputs (torque wheels) and two outputs (Kaman sensors). The inputs are random forces with unit strength. The process noise is set at approximately 23% of the input influence and the measurement noise about 10% of the output, both in the standard deviation ratio. The sampling rate is 33.3 Hz (0.03 s). The order of the adaptive transversal filter is set at 100. The identified modal frequencies and damping factors with the corresponding number of data processed are listed in Table 2. Although more data than the previous example are needed to identify the modal frequencies and dampings, the results are fairly accurate even in the presence of repeated modes.

When a system has repeated eigenvalues, the mode shapes of the repeated frequency are not unique even though they are normalized, as described in the previous section. For a specific repeated frequency, any linear combination of the identified mode shapes may be used as a mode shape. This does not impose any problem if the system identification and state estimation is conducted for control purpose because a state feedback controller design only requires a set of state-space model and filter gain, regardless of what coordinate the model might refer to. However, the Markov parameters of the system are unique because they are independent of the state coordinate. Hence, the quality of the identified model can be evaluated by comparing the reconstructed system Markov parameters ($\hat{C}\hat{A}^{i-1}\hat{B}$) with that of the original system ($CA^{i-1}B$). Figure 7 shows the comparison of one element of the Markov parameter matrices after 15,000 data processed. They agree quite well in the beginning and become worse as the power index number i increases. Another alternative for evaluating the quality of the identified model is to compare the singular values of the corresponding transfer function matrix over an interested frequency range with that of the original system. Figures 8 show such a comparison, where the identified model is obtained after 15,000 data processed. A good agreement is obtained with some deviation caused by noise-induced errors in the identified frequencies, dampings, and mode shapes. To examine the quality of the identified filter gain, an element of the reconstructed filter Markov parameter matrices [the (1,1) element of $\bar{C}\bar{A}^{i-1}\bar{A}\bar{K}$, $i = 1, \ldots, 100$] is compared to the true one, which is shown in Fig. 9 for an identified filter gain obtained after 15,000 data processed. Figures 7–9 show that the identified model including an optimal filter in this example is fairly good after 15,000 data processed.

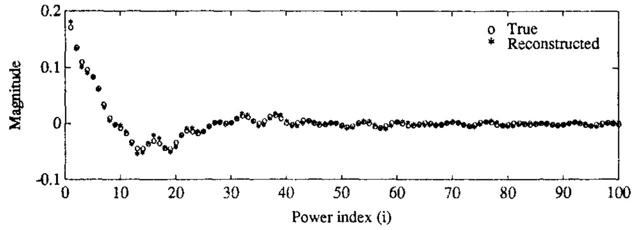


Fig. 9 Comparison of a reconstructed filter Markov parameter sequence [the (1,1) element of $\bar{C}\bar{A}^{i-1}\bar{A}\bar{K}$] and its true value after 15,000 data processed.

Concluding Remarks

The main contribution of this paper is the derivation of an algorithm that can simultaneously identify a system state-space model and an optimal Kalman filter gain from experimental data. This algorithm provides a great advantage if the system identification is conducted for control purpose because a state estimator is readily constructed. In contrast with most existing methods in modal parameter identification of structures where a great majority of them use a deterministic approach, this stochastic method takes the influence of both process and measurement noises into consideration. It reduces the effect of noise on the identified parameters, especially the process noise that is usually neglected by deterministic methods, and provides better results. Furthermore, the algorithm takes the advantage of the recursive least-squares filter, which can continually improve the estimated system and filter models for state estimation. This feature makes it attractive in designing an adaptive controller for control of flexible space structures.

Appendix A: Equivalent Transformation

Any two equivalent systems, $[A_1, B_1, C_1]$ and $[A_2, B_2, C_2]$, where both A_1 and A_2 have distinct eigenvectors, can be transformed to the same triplet $[A_n, B_n, C_n]$ by a certain equivalent transformation. In the following, the proof is given.

Proof: After diagonalizing the system matrix and normalizing the output or input matrix, respectively, $[A_1, B_1, C_1]$ and $[A_2, B_2, C_2]$ become $[\Lambda_1, B_{n1}, C_{n1}]$ and $[\Lambda_2, B_{n2}, C_{n2}]$, where

$$\Lambda_1 = D_1^{-1}A_1D, \quad B_{n1} = D_1^{-1}B_1, \quad C_{n1} = C_1D_1 \tag{A1}$$

$$\Lambda_2 = D_2^{-1}A_2D, \quad B_{n2} = D_2^{-1}B_2, \quad C_{n2} = C_2D_2 \tag{A2}$$

where D_1 and D_2 are the equivalent transformation matrices for the corresponding set.

Since $[A_1, B_1, C_1]$ and $[A_2, B_2, C_2]$ are equivalent, there also exists a nonsingular matrix P such that

$$A_1 = P^{-1}A_2P, \quad B_1 = P^{-1}B_2, \quad C_1 = C_2P \tag{A3}$$

The similarity transformation will not change the eigenvalues; therefore, A_1 and A_2 have the same eigenvalues. This means that Λ_1 and Λ_2 are identical assuming that the eigenvalues have been sorted in the same order. Hence, from Eqs. (A1–A3), one can have

$$\begin{aligned}\Lambda_1 &= D_1^{-1}A_1D_1 = D_1^{-1}P^{-1}A_2PD_1\\ &= \Lambda_2 = D_1^{-1}A_2D_2 = A_n\end{aligned} \tag{A4}$$

From Eq. (A4), because both PD_1 and D_2 diagonalize A_2, the following relation must hold:

$$PD_1 = D_2K_c \tag{A5}$$

where K_c is some nonsingular constant diagonal matrix. Therefore, from Eqs. (A1–A5) one obtains

$$\begin{aligned}C_{n1} &= C_1D_1 = C_1P^{-1}D_2K_c = C_2PP^{-1}D_2K_c\\ &= C_{n2}D_2^{-1}PP^{-1}D_2K_c = C_{n2}K_c\end{aligned} \tag{A6}$$

Since K_c is a constant diagonal matrix, C_{n1} and C_{n2} are scaled versions to each other. However, since both matrices have been normalized in the same way, the only possible solution for K is the identity matrix, which means $C_{n1} = C_{n2}$. Similarly, B_{n1} and B_{n2} can be proved to be identical. As a result, the unique set $[A_n, B_n, C_n]$ is obtained.

Appendix B: True and Identified Matrices

The true state-space model and the steady-state Kalman filter gain and their estimated versions using 5000 data are shown here for comparison:

$$A_m = \text{diag}\left\{\begin{bmatrix} 0.9856 & 0.1628\\ -0.1628 & 0.9856\end{bmatrix} \begin{bmatrix} 0.8976 & 0.4305\\ -0.4305 & 0.8976\end{bmatrix} \begin{bmatrix} 0.8127 & 0.5690\\ -0.5690 & 0.8127\end{bmatrix}\right\}$$

$$B_m = [0.0011 \quad 0.0134 \quad -0.0016 \quad -0.0072 \quad 0.0011 \quad 0.0034]^T$$

$$C_m = \begin{bmatrix} 1.5119 & 0. & 2.0000 & 0. & 1.5119 & 0.\\ 1.3093 & 0. & 0. & 0. & -1.3093 & 0.\end{bmatrix}$$

$$K_m = \begin{bmatrix} 0.0604 & 0.0279 & 0.0471 & 0.0146 & 0.0132 & 0.0055\\ -0.0648 & 0.0366 & -0.0059 & 0.0143 & -0.0162 & -0.0011\end{bmatrix}^T$$

where m denotes modal form and A_m is a block diagonal matrix

$$\hat{A}_m = \text{diag}\left\{\begin{bmatrix} 0.9793 & 0.1605\\ -0.1605 & 0.9793\end{bmatrix} \begin{bmatrix} 0.9006 & 0.4255\\ -0.4255 & 0.9006\end{bmatrix} \begin{bmatrix} 0.7831 & 0.5731\\ -0.5731 & 0.7831\end{bmatrix}\right\}$$

$$\hat{B}_m = [0.0016 \quad 0.0113 \quad -0.0039 \quad -0.0073 \quad 0.0030 \quad 0.0038]^T$$

$$\hat{C}_m = \begin{bmatrix} 1.5207 & 0. & 1.9919 & 0. & 1.5314 & 0.\\ 1.2988 & 0.0247 & 0.0704 & -0.1658 & -1.2485 & -0.3100\end{bmatrix}$$

$$\hat{K}_m = \begin{bmatrix} 0.0618 & 0.0233 & 0.0436 & 0.0083 & 0.0078 & 0.0080\\ 0.0698 & 0.0263 & -0.0052 & 0.0240 & -0.0278 & -0.0076\end{bmatrix}^T$$

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